# Shape Gradients & Tolerances

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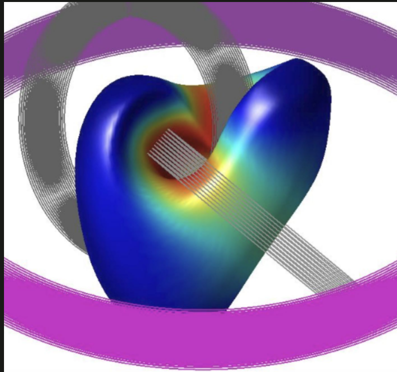


# Motivation

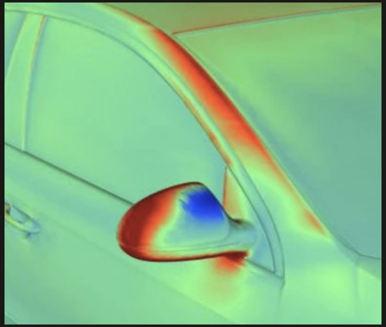
Create algorithms to compute shape gradients and tolerances to be applied in stellarator optimization and design

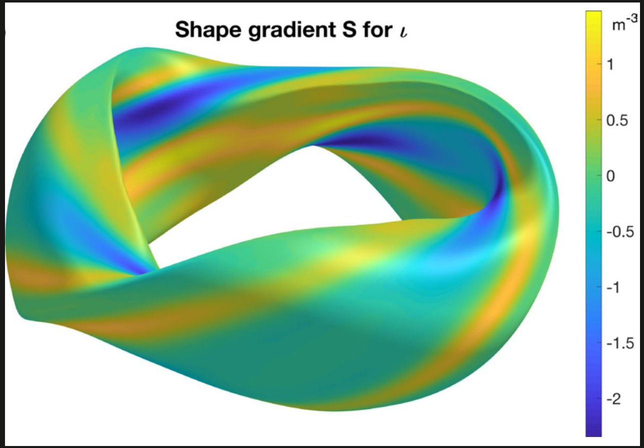
- Which configuration would maximize tolerances for crucial parameters?

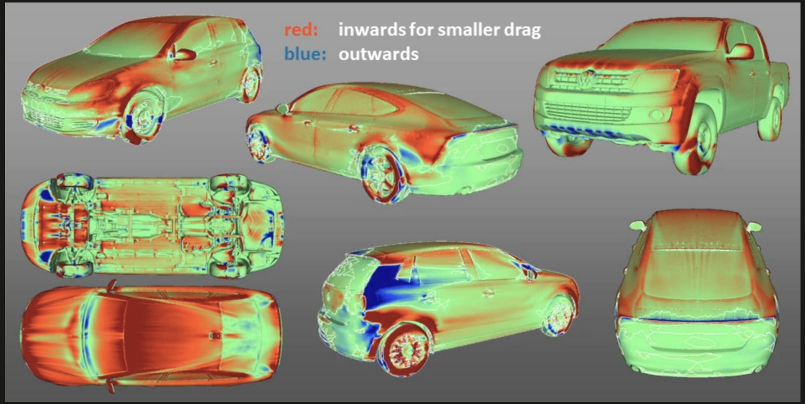
- Where should we put coil supports?
- Where should magnetic material be located?

# Why Is It Useful?

- Automotive industry (drag sensitivity)
- Aerospace industry (structural integrity)
- Civil engineering (structural optimization)
- Stellarator optimization and design!





# What Is Shape Gradient?

For coil functionals:

$$\delta f = \sum_k \int d\ell \, \boldsymbol{S}_k \cdot \delta \boldsymbol{r},$$

$$\updownarrow$$

$$\int_0^{2\pi} d\vartheta \left| \frac{d\boldsymbol{r}}{d\vartheta} \right| \frac{\partial \boldsymbol{r}}{\partial p_j} \cdot \boldsymbol{S} = \frac{\partial f}{\partial p_j}$$

- Gives the local differential contribution to some sclar figure of merit caused by normal displacements
- Can be computed from derivatives w.r.t. shape parameters for appropriate figures of merit
- Provides spatially local information

# What Is Tolerance?

- Can be computed from shape gradients
- Local / uniform

$$T_k(\ell) = \frac{w_k(\ell)\,\Delta f}{\sum_{k'} \int \mathrm{d}\ell'\, w_{k'}(\ell')|\boldsymbol{S}_{k'}(\ell')|}$$

$$w_k = |\boldsymbol{S}_k|^{-\alpha}$$

$$w_k = 1$$

# Problem Description

$$x(l) = \sum_{m=0}^{\text{order}} x_{c,m} \cos(2\pi lm) + \sum_{m=1}^{\text{order}} x_{s,m} \sin(2\pi lm)$$

$$y(l) = \sum_{m=0}^{\text{order}} y_{c,m} \cos(2\pi lm) + \sum_{m=1}^{\text{order}} y_{s,m} \sin(2\pi lm)$$

$$z(l) = \sum_{m=0}^{\text{order}} z_{c,m} \cos(2\pi lm) + \sum_{m=1}^{\text{order}} z_{s,m} \sin(2\pi lm)$$

$$dofs = [x_{c,0}, x_{s,1}, x_{c,1}, \cdots, x_{s,\text{order}}, x_{c,\text{order}}, y_{c,0}, y_{s,1}, y_{c,1}, \cdots]$$

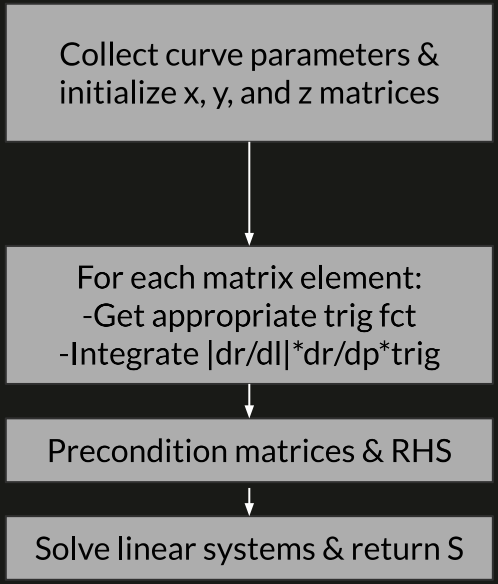
$$\int_0^1 \left|\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}l}\right| \frac{\partial \mathbf{r}}{\partial p_j} \cdot \mathbf{S} \mathrm{d}l = \frac{\partial f}{\partial p_j}$$

$$T(l) = \frac{w(l)\,\Delta f}{\int w(l')|S(l')|dl'}$$

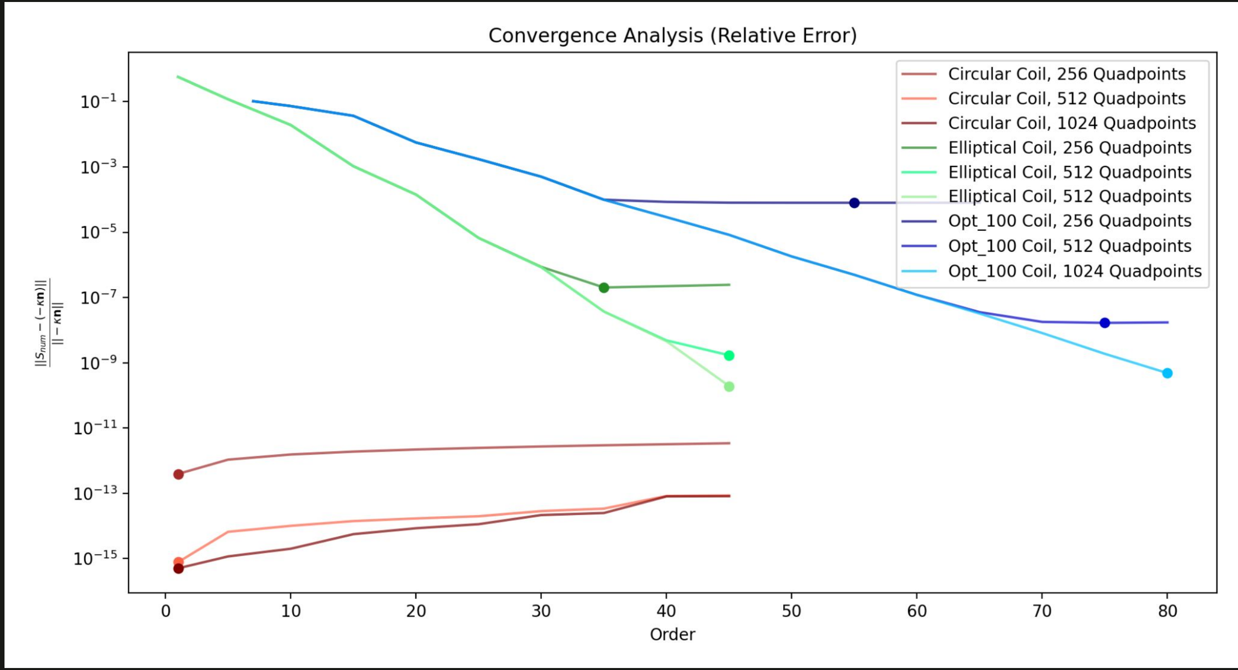
- Given a curve and a figure of merit, compute the shape gradient (constrained vs unconstrained)
- Using the shape gradient, evaluate uniform and local tolerances

# Pseudocode (for Unconstrained Curve)

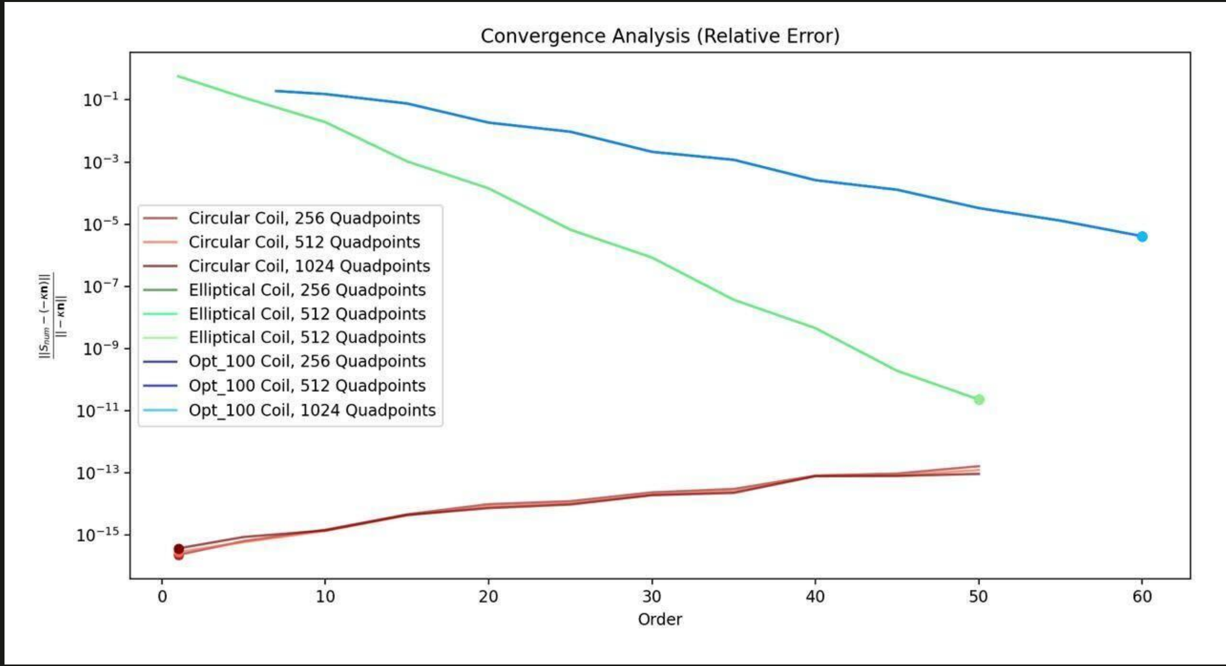
```
evaluate SHAPE_GRADIENT(curve, df_dp):
    - Initialize matrices A_x, A_y, A_z with zeros
    - Extract number of dofs (D),
      number of quadpoints (N_quad), and order (N_ord)
    - Get derivatives w.r.t. arclength (dr_dl)
      and magnitude of dr_dl (abs_dr_dl)
    - Get derivatives w.r.t. coefficients (dr_dp)

    - For each i, j in range(D // 3):
        * Calculate trig function (cosine or sine based on j)
        * Compute matrix elements A_x, A_y, A_z by integrating
          abs_dr_dl*dr_dp*trig over quadrature points

    - Precondition A_x, A_y, A_z and RHS (df_dp)

    - Try solving the linear systems for S_x, S_y, S_z
    - Concatenate and return the solution S
```



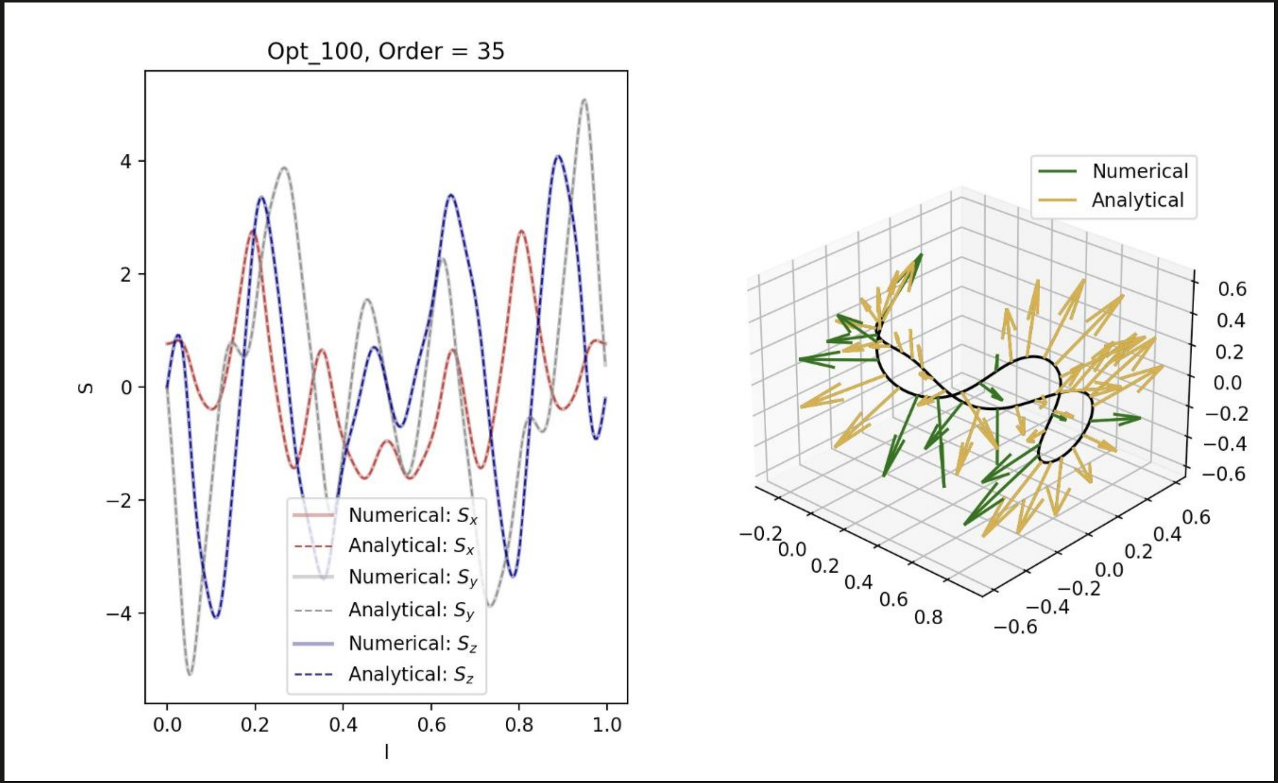
# Performance: f=CurveLength
# Algorithm Using Interpolation + Adaptive Quadrature



# Performance: f=CurveLength
# Algorithm Using Periodicity of Curve Representation



# Performance: f=CurveLength



# Performance: f=NonQSRatio